

A TURBULENT MODEL FOR THE INTERSTELLAR MEDIUM. II. MAGNETIC FIELDS AND ROTATION

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Abstract

We present results from two-dimensional numerical simulations of a supersonic turbulent flow with parameters characteristic of the interstellar medium at the 1 kpc scale in the plane of the galactic disk, incorporating shear, thresholded and discrete star formation (SF), self-gravity, rotation and magnetic fields. A test of the model in the linear regime supports the results of the linear theory of Elmegreen (1991a). At low shear, a weak azimuthal magnetic field stabilizes the medium by opposing collapse of radial perturbations, while a strong field is destabilizing by preventing Coriolis spin-up of azimuthal perturbations (magnetic braking). At high shear, azimuthal perturbations are sheared into the radial direction before they have time to collapse, and the magnetic field becomes stabilizing again.

In the fully nonlinear turbulent regime, while some results of the linear theory persist, new effects also emerge. The production of turbulent density fluctuations appears to be affected by the magnetic field as in the linear regime: moderate field strengths cause a decrease in the time-integrated star formation rate, while larger values cause an increase. A result not predicted by the linear theory is that for very large field strengths, a decrease in the integrated SFR obtains again, indicating a “rigidization” of the medium due to the magnetic field. Other exclusively nonlinear effects are: a) Even though there is no dynamo in 2D, the simulations are able to maintain or increase their net magnetic energy in the presence of a seed uniform azimuthal component. b) A well-defined power-law magnetic spectrum and an inverse magnetic cascade are observed in the simulations, indicating full MHD turbulence. Thus, magnetic field energy is generated in regions of SF and cascades up to the largest scales. c) The field has a slight but noticeable tendency to be aligned with density features. This appears to be as much a consequence of the gas pushing on the magnetic field as due to constraints on gas motions because of the presence of the magnetic field. d) A “pressure cooker” effect is observed in which the magnetic field prevents HII regions from expanding freely, as in the recent results of Slavin & Cox (1993). e) The orientation of the large-scale azimuthal field appears to follow that of the large-scale Galactic shear. f) A tendency to exhibit *less* filamentary structures at stronger values of the uniform component of the magnetic field is present in several magnetic runs. Possible mechanisms that may lead to this result are discussed. g) For fiducial values of the parameters, the flow in general appears to be in rough equipartition between magnetic and kinetic energy. There is

no clear domination of either the magnetic or the inertial forces. h) A median value of the magnetic field strength within clouds is $\sim 12\mu\text{G}$, while for the intercloud medium a value of $\sim 3\mu\text{G}$ is found. Maximum contrasts of up to a factor of ~ 10 are observed.

Subject headings: ISM: clouds – ISM: evolution – ISM: magnetic fields – ISM: structure – instabilities – turbulence.

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1. INTRODUCTION

In Paper I of this series (Vázquez-Semadeni, Passot & Pouquet 1995; see also Passot, Vázquez-Semadeni & Pouquet 1994), we presented two-dimensional (2D) numerical simulations attempting to model the Interstellar Medium (ISM) in the plane of the Galaxy at the kpc scale. The model incorporates self-gravitating hydrodynamics, parameterized radiative cooling, diffuse heating, and a prescription for modeling star formation (SF). Some of the first results obtained from the numerical simulations were the “slaving” of the temperature and thermal pressure to the density in the absence of thermal instabilities due to the short thermal time scales, the existence of a self-sustained cycle in which the stellar energy input to the ISM is enough to maintain the turbulence (with an efficiency of 0.05 %), the formation of clouds and cloud complexes mainly through collisions of gas streams (turbulent ram pressure), and a large scatter in the virial ratios of cloud energies, although nearly-virial clouds exhibited a tendency to live longer, explaining their observed overabundance.

Among the significant shortcomings of the model of Paper I, the most notorious one is an excessive cloud temperature ($\gtrsim 1000$ K), and consequently, too low a density contrast ($\rho_{\max}/\rho_{\min} \sim 50$). Additionally, two of the most obvious omissions are magnetic fields and rotation of the Galactic disk. In the present paper we present improvements of the model along both lines: we first introduce a different diffuse heating mimicking the effect of shielding against background UV radiation; this allows the flow to reach more realistic cloud temperatures (a few hundred K) and density contrasts (~ 1000). We then extend the model by incorporating both magnetic fields and rotation.

The magnetic field in the ISM is ubiquitous, although its precise dynamical effects (e.g. Troland 1990) and topology (e.g. Trimble 1990; Heiles et al. 1993) are still a matter of active research and debate. Some fundamental unanswered questions concerning the field are:

- 1) Is the field of primordial origin or is it continually generated and dissipated in the Galaxy? In the latter case, what are the generation/amplification mechanisms? (e.g. Zweibel 1987; Wiebelsky & Krause 1993; Tajima, Cable, Shibata & Kulsrud 1992)?
- 2) What is the amplitude ratio between its uniform and fluctuating components, and what are the typical scales of the latter?
- 3) Is the field fully turbulent, or does it consist simply of a superposition of weakly interacting waves?
- 4) What is its relative importance in the global dynamics and energetics of the ISM and in cloud formation and support (e.g., Falgarone & Puget 1986, Troland 1990, Pudritz & Gómez de Castro 1991; Elmegreen 1991a; McKee et al. 1993)?
- 5) Does the field strength correlate with gas density (Mouschovias, 1976a, b; García-Barreto et al. 1987; Crutcher, Kazes & Troland 1987; Myers & Goodman 1988a)?
- 6) Does the field orientation correlate with density features (cloud shapes and elongations) (Goodman 1991; see also Heiles et al. 1993)?
- 7) Is there a tendency for equipartition between kinetic, magnetic and gravitational

energies in molecular clouds (MCs) as observations seem to indicate (Myers & Goodman 1988a,b)?

8) Is the Alfvén speed the typical velocity of propagation of disturbances in the ISM (Falgarone & Puget 1986; Myers & Goodman 1988a,b)?

Concerning rotation, it has long been known that it stabilizes the gravitational collapse of structures in the Galactic disk (Chandrasekhar, 1961; Toomre 1964; Goldreich & Lynden-Bell 1965). Two main effects are present: the conversion of compressive motions into shearing ones by the Coriolis force, and the “restoring force” against radial motions arising from the interplay between the radial component of gravity and the centrifugal force. Recently, a combined instability analysis of a self-gravitating, rotating flow with heating and cooling and magnetic fields has been performed by Elmegreen (1991a, 1994). In the turbulent case, the main mode of cloud formation identified in Paper I—that arising from random turbulent compressions—may also be inhibited by rotation.

In the present paper, we present two-dimensional numerical simulations aimed at investigating these problems. Our simulations are limited by two facts. First, since the simulations reported are 2D, they cannot produce a dynamo, which is intrinsically a 3D effect. However, this has the advantage that other mechanisms of amplification of the magnetic field can be isolated and identified. Second, the numerical code used is pseudospectral, with periodic boundary conditions. The latter imply that the centrifugal and radial gravitational forces, which depend on the galactocentric distance, cannot be included in the model. However, as shown in §2.2, all the relevant aspects of the dynamics are included in the Coriolis force and the large scale shear, so that the omission turns out to be inconsequential.

The plan of the paper is as follows: §2 presents the equations of the model and discusses the values of the parameters; §3 describes the results of simulations in the linear regime, and §4 discusses the nonlinear behavior of the model, stressing in particular the differences between the two regimes. In §5 we discuss the general behavior of a simulation with fiducial values of the parameters. Finally, §6 contains a brief summary and a discussion of the reaches and limitations of the current simulations.

2. THE MODEL

2.1. Equations

As in Paper I, we use a single-fluid approach to represent the ISM with several source terms in order to model radiative cooling, large-scale shear, and stellar and diffuse heating. The computations solve the nondimensionalized equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \mu \nabla^2 \rho, \quad (1a)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla P}{\rho} - \nu_8 \nabla^8 \mathbf{u} - \left(\frac{J}{M_a}\right)^2 \nabla \phi + \frac{1}{\rho} (\nabla \times \mathbf{B}) \times \mathbf{B} - 2\Omega \times \mathbf{u}, \quad (1b)$$

$$\frac{\partial e}{\partial t} + \mathbf{u} \cdot \nabla e = -(\gamma - 1)e \nabla \cdot \mathbf{u} + \kappa_T \frac{\nabla^2 e}{\rho} + \Gamma_d + \Gamma_s - \rho \Lambda, \quad (1c)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nu_8 \nabla^8 \mathbf{B}, \quad (1d)$$

$$\nabla^2 \phi = \rho - 1, \quad (1e)$$

in two dimensions [on the (x, y) -plane with $\partial/\partial z = 0$] using periodic boundary conditions. As usual, ρ is the density, \mathbf{u} is the fluid velocity, e is the internal energy per unit mass, P is the thermal pressure, \mathbf{B} is the magnetic field, Ω is the angular velocity of the rotation, and ϕ is the gravitational potential. Furthermore, we will frequently make use of the number density $n \equiv \rho/m_H$, where m_H is the mass of the hydrogen atom. We recover identically the model of Paper I by setting $\mathbf{B} \equiv 0$ and $\Omega \equiv 0$ in the above equations, although some of the parameter values may have changed, and the diffuse heating is different (see below, §2.2).

We use an ideal-gas equation of state $P = (\gamma - 1)\rho e$, where $\gamma = c_p/c_v$ is the ratio of specific heats at constant pressure and volume, respectively. We take $\gamma = 5/3$. The temperature is related to the internal energy by $e = c_v T$. The variables are normalized to characteristic values ρ_o, u_o, L_o, T_o and B_o , given in Table 1. We refer the reader to Paper I for a thorough discussion of most of the parameters and model terms. Section 2.2 below describes those that are new or have changed for the present paper. The physical dimension corresponding to the side of the integration box is L_o . The nondimensional parameters resulting from the normalization are: the Mach number $M_a = u_o/c_o$, where $c_o = (\gamma k T_o/m_H)^{1/2}$ is the adiabatic speed of sound at the normalizing temperature T_o ; the Jeans number $J = L_o/L_J$, giving the number of Jeans' lengths $L_J = (\pi c_o^2/G\rho_o)$ in the simulation. The thermal diffusivity is denoted κ_T .

The pseudospectral scheme used here introduces no numerical viscosity, since in Fourier space the MHD equations become a set of coupled ordinary differential equations for the Fourier amplitudes. Thus, we include explicit dissipation terms in the equations for \mathbf{u} and \mathbf{B} . A hyperviscosity scheme with a ∇^8 operator is used, which confines viscous effects to the smallest resolved scales. Indeed, for a fixed amount of dissipation at a given small scale, the higher power of the Laplacian allows for a smaller effective dissipation at the large scales. This technique is widespread in the fluid dynamics community (see, for example, McWilliams 1984; Babiano et al., 1987). Note that the diffusion coefficients for the velocity and magnetic fields are taken to be the same and equal to ν_8 .

In addition, a mass diffusion $\mu \nabla^2 \rho$ is included in the continuity equation in order to smooth out the density gradients, thus allowing the simulations to reach higher rms Mach numbers. The effects of this term are discussed at length in Paper I.

Note that the kinematic viscosity and the mass diffusion coefficients are chosen so that velocity and density discontinuities are spread out a few pixels, guaranteeing that they can be resolved. Also, note that heat generated by the kinetic and magnetic dissipations is not included in the calculations because it is negligible compared to the stellar and diffuse heating.

Concerning self-gravity, we note that equation (1e) is a modified Poisson

equation appropriate for infinite media, and its application to pseudospectral simulations with periodic boundary conditions has been discussed in detail by Alecian & Léorat (1988). Essentially, this equation represents the self-gravity of the density *fluctuations*.

Further details on the pseudospectral integration technique we use can be found in Paper I. The theoretical bases on which it rests can be found in Canuto et al. (1988). As is standard for turbulence simulations, the initial conditions for all variables are Gaussian fluctuations with random phases. The typical scale size of the initial density, temperature and velocity fluctuations is 1/8 of the integration box. The typical scale size of the initial magnetic field fluctuations is specified by the parameter \mathbf{k}_B , discussed in §2.3.2 below.

A few words are in order concerning the choice of units (see Table 1). The unit of length L_o , equal to 1 kpc in physical units, is 2π in code units. Derived units which involve this length also contain factors of 2π . Also, the unit of velocity u_o is chosen equal to the speed of sound c_o at the unit of temperature T_o . The fundamental density unit in the code has units of a volume density. Finally, we note that the time step in a typical run varies in the range .01–.001 in units of 1.3×10^7 years.

2.2. Model Terms

In this section we first give a brief summary of the forms used for the model terms. Only model terms that are new or have changed from Paper I are discussed in detail here. A detailed description of all other terms can be found in Paper I. Numerical values of their associated parameters are given in Table 1.

2.2.1. Diffuse heating Γ_d

As pointed out in the Introduction, the simulations presented in Paper I had one important shortcoming: the densest regions that can be associated with clouds still had temperatures much larger ($\sim 1,500$ K) than actual cloud temperatures (10–100 K) together with too low a density ($\rho_{\max} \sim 10\rho_o$). This problem was mainly due to the form of the diffuse heating used in that paper, namely a constant, uniform heat source. This form of the diffuse heating produced realistic temperatures for the Intercloud Medium (ICM) (although see the discussion in Paper I concerning the uncertainties surrounding the temperature of this $n \sim 1 \text{ cm}^{-3}$ gas), but it caused the medium to be too “hard” (resistive to compression), thus both reducing the density contrast ($\rho_{\max}/\rho_{\min} \sim 50$), and keeping the cloud temperatures too high. Physically, if most of the diffuse background energy is in low-energy UV photons, then it is known that clouds can shield their interiors from this radiation if their column densities are high enough (Franco & Cox 1986). Conversely, if most of the background energy is in energetic particles, then the heating rate per unit mass should be rather insensitive to the cloud density. As a compromise between these two possibilities, we now adopt a new diffuse heating function of the form

$$\Gamma_d(\mathbf{x}, t) = \Gamma_o(\rho/\rho_{\text{IC}})^{-\alpha}$$

where ρ_{IC} is a typical density of the ICM and α is a free parameter, which in general we take such as to give a weak dependence of Γ_d on ρ . The factor ρ_{IC}^{-1}

in the density is introduced so that an equilibrium temperature (between diffuse heating and radiative cooling) of 10^4 K is obtained at the density of the ICM. This form of Γ_d is not intended to represent any realistic physical dependence, since self-shielding depends on column density, which is a non-local cloud property, and thus extremely costly to implement in our code. Instead, the adopted form provides a smooth density dependence. An experiment with a threshold criterion for turning off the diffuse heating above a critical density was presented in Paper I, but it was found that it introduced spurious oscillations of the density, temperature and pressure distributions in clouds. The fiducial values we adopt for the parameters of the diffuse heating are $n_{\text{IC}} \equiv \rho_{\text{IC}}/m_{\text{H}} = 0.2 \text{ cm}^{-3}$ and $\alpha = 1/2$. A discussion of the effects of variations in α (which is reflected on an effective polytropic exponent γ_{eff} [§3.1]) on the stability of the model is given in §3.2.

2.2.2. *Stellar Heating* Γ_s

We model local stellar heating due to massive stars by means of a threshold algorithm: a heating center a few pixels across (with a Gaussian profile) is turned on wherever the density exceeds a threshold value ρ_c and $\nabla \cdot \mathbf{u} < 0$. Once a “star” is turned on, it stays on for a time $\Delta t = 6 \times 10^6$ yr, typical of the lifetime of OB stars. Note that once a star is turned on, it remains fixed with respect to the numerical grid.

As described in Paper I, the star formation rate (SFR) is computed in the simulations as the fraction of pixels that reach ρ_c per unit time.

2.2.3. *Cooling* Λ

We use the parameterization of the radiative cooling functions of Raymond & Cox (1976) and Dalgarno & McCray (1972), as employed by Rosen et al. (1993) and Rosen & Bregman (1994):

$$\Lambda = \begin{cases} 0 & 0 \leq T < 100 \text{ K} \\ \Lambda_1 T^2 & 100 \text{ K} \leq T < 2000 \text{ K} \\ \Lambda_2 T^{1.5} & 2000 \text{ K} \leq T < 8000 \text{ K} \\ \Lambda_3 T^{2.867} & 8000 \text{ K} \leq T < 10^5 \text{ K} \\ \Lambda_4 T^{-0.65} & 10^5 \text{ K} \leq T < 4 \times 10^7 \text{ K} \end{cases}$$

Similarly to Paper I, the cooling and heating rates are decreased by a factor of 7 with respect to realistic values for numerical reasons, in order to reduce the stiffness of the resulting system. This, however, is not expected to affect the dynamics as their characteristic time scales are still much shorter than dynamical time scales (by factors between 10 and 10^4). As discussed in Paper I, the shorter thermal time scales have the interesting consequence that the system is capable to reach thermal equilibrium between cooling and diffuse heating in times much shorter than the characteristic time for turbulence-induced density fluctuations to occur. The flow thus behaves as a polytropic gas with an effective polytropic exponent given by the condition of thermal equilibrium (§3.1). This behavior is preserved as long as the thermal rates are much faster than the dynamical rates, so that the equilibrium temperature can be reached before the density changes appreciably,

as is the case of our simulations. Even in the case of strong heating due to the localized stellar heating, thermal equilibrium between the latter and the cooling is achieved virtually instantaneously. In this case a strong pressure imbalance with the surroundings exists, due to the pointwise nature of the stellar heating, producing expanding motions. Thus, the dynamical effects of the stellar heating are preserved in spite of the reduced thermal rates we use.

It should also be pointed out that in our simulations the gas temperature never exceeds 3.5×10^4 K, since heating due to massive-star winds and supernovae is not considered (see §6.2 for a discussion of limitations of the simulations). Thus, the strong thermal instability above 10^5 K in the cooling function is never reached in practice, and is included only for completeness. Work including such heating mechanisms is in progress (Gazol et al. 1995).

Finally, the lower cutoff of the cooling is discussed at length in Paper I, although note that here we have extended it to 100 K, down from 300 K in Paper I. This limit still does not pose significant numerical difficulties, yet it allows the simulations to achieve more realistic cloud temperatures.

2.2.4. Rotation

We assume that our integration domain is located roughly at the solar circle, rotating about the Galactic center at a radius R_o and with angular speed Ω_o , with the x - and y -coordinates in the model corresponding to the azimuthal and radial directions, respectively. In this rotating frame one must consider the Coriolis force $-2\Omega_o \times \mathbf{u}$, the centrifugal force $\Omega_o^2 R \hat{\mathbf{e}}_y$ and the radial gravitational force \mathbf{g}_R of the total mass contained within radius R . In the actual Galactic disk, the latter two forces exactly balance each other at every radius R , and $\mathbf{g}_R = -\Omega^2(R) R \hat{\mathbf{e}}_y$. This balance is at the origin of the differential rotation of the Galactic disk and its associated shear. In a rotating frame characterized by a single angular velocity Ω_o , the total fluid velocity is the sum of the shear $\mathbf{u}_s = u_s \hat{\mathbf{e}}_x$ and a turbulent fluctuation \mathbf{u}_{turb} . Then, at radius R , the difference between the centrifugal force due to the rotation velocity of the frame and the radial gravitational force is, to first order, $-2\Omega_o(d\Omega/dR)_{R_o} R y \hat{\mathbf{e}}_y$, and this excess is balanced by the Coriolis force acting on the shear velocity, which gives $u_s = -R(d\Omega/dR)y$.

The simulation of this interaction of forces and the resulting shear can be accomplished at two equivalent levels. At a more fundamental level, the radial gravitational force can be included in the equations, with an explicit R - (or, equivalently, a y -) dependence. At a less physical, but less numerically costly level, the resulting shearing profile can be simply imposed on the velocity field, without introducing the radial centrifugal force in the equation of motion. We have chosen the latter option. Note, however, that due to the periodic boundary conditions we use, we cannot introduce a monotonic function of radius. Instead, we need to introduce a periodic function in the integration box, the simplest of which is a sinusoidal profile of period one of the form

$$u_x(y) = A_o \sin \frac{2\pi y}{L_o}. \quad (2)$$

This is trivially accomplished by fixing the first Fourier mode of the velocity to a constant. We choose the shear to have period one in order to approximate in as large as possible a fraction of the box the monotonic character of the actual shear in the Galactic disk.

It should be kept in mind that, as given by the above expression, the shear has the correct sign of $d\Omega/dR$ (i.e., Ω decreasing outwards) in only half of the integration box (the upper and lower quarters of the box), and the opposite sign in the central half of the box. This has dynamical consequences which will be discussed in §3.2.

Toh, Ohkitani & Yamada (1991) have developed an algorithm for simulating incompressible 2D flows with linear shear profiles with pseudospectral techniques (see also Feireisen, Reynolds & Ferziger 1981 for the compressible case). In MHD, a special choice of gauge for the magnetic potential can be used to accommodate shear profiles (Brandenburg et al. 1995). Here, for simplicity, we have adopted the above sinusoidal velocity profile.

2.3. Parameters

A detailed description of the criteria used to select most parameter values is given in Paper I. Table 1 gives a summary of the parameters' meanings and fiducial values, both in physical units and in nondimensional code units. In this section we only discuss the values and criteria for the new parameters related to the magnetic field and the Coriolis force.

2.3.1 Rotation

We adopt a rotational speed of 250 km s^{-1} (see, e.g., Shore 1989), implying $\Omega = 3.14 \times 10^{-8} \text{ yr}^{-1}$, which, in units of the code becomes $\Omega = 0.41$.

2.3.2. Magnetic field

Current views picture the Galactic magnetic field along the Galactic plane in the solar neighborhood as having a nearly azimuthal uniform component of strength $B_u \sim 1.5\mu\text{G}$, and a turbulent component $B_t \sim 5\mu\text{G}$ (e.g. Rand & Kulkarni 1989, Wielebinski & Krause 1993). We choose a unit of the magnetic field strength such that $v_A = u_o$. At the unit of density for the code ($\rho_o = 1 \text{ cm}^{-3}$), this corresponds to $B_o = 5\mu\text{G}$. Note that, in the nondimensional code units the Alfvén speed is given by $v_A^2 = B^2/\rho$.

Finally, as mentioned in §2.1, another initial parameter of the simulations is \mathbf{k}_B , the characteristic wavenumber of the initial magnetic field fluctuations. We adopt a fiducial value $\mathbf{k}_B = 1$, although the effect of varying this parameter is described in §4.2.

3. LINEAR EVOLUTION

As will be seen in §4, the fully nonlinear behavior of the model is extremely complex. In order to appropriately interpret the various processes at play and to distinguish between linear and nonlinear effects, we first discuss the linear evolution of the model. The discussion in this and the following sections relies heavily on the results of nearly 70 runs we performed, which we used to explore the effects of variations in the parameters (low resolution runs, typically 128^2 grid points) and

to analyze the structure and evolution of the model with the fiducial values of the parameters (run 28, with 512^2 grid points). A summary of the parameters of the runs referred to in this paper is given in Table 2.

3.1. Equilibrium state

For the linear evolution runs, the system is started from an equilibrium state with uniform density $n_o = 1 \text{ cm}^{-3}$, velocity field given by eq. (2), uniform temperature and a uniform magnetic field in the x (“azimuthal”) direction. Several cases with various values of the initial uniform magnetic field have been analyzed. The initial value of the temperature is that corresponding to equilibrium between cooling and the diffuse heating, and is thus a function of the density exponent α in the expression for Γ_d (§2.2). Indeed, setting $\rho\Lambda = \Gamma_d$ gives the following equilibrium values for the temperature and thermal pressure of the flow:

$$T_{\text{eq}} = \left[\frac{\Gamma_o \rho_{\text{IC}}^\alpha}{\Lambda_i \rho^{1+\alpha}} \right]^{1/\beta_i} \quad P_{\text{eq}} = \frac{\rho T_{\text{eq}}}{\gamma} = \frac{\rho^{\gamma_{\text{eff}}}}{\gamma} \left[\frac{\Gamma_o \rho_{\text{IC}}^\alpha}{\Lambda_i} \right]^{1/\beta_i}, \quad (3)$$

where $\gamma_{\text{eff}} = 1 - (1 + \alpha)/\beta_i$, and β_i and Λ_i are respectively the exponent and coefficient of the temperature in the i -th range of the cooling function. Thus, the initial temperature listed in Table 2 for each run, if different from 10^4 K , is the equilibrium temperature appropriate for the value of α in that run. Note that runs in the fully nonlinear regime, discussed in §4, always start at $T = 10^4 \text{ K}$. For small perturbations about this equilibrium state, the effective speed of sound c_{eff} satisfies, in nondimensional units, $c_{\text{eff}}^2 = \gamma_{\text{eff}} P_{\text{eq}} / \rho_{\text{eq}} = (\gamma_{\text{eff}} / \gamma) T_{\text{eq}}$.¹

3.2. Dispersion relation and stability criteria

The original gravitational instability analysis of Jeans (1902) has been continually extended by a number of authors to include additional processes, such as rotation (Chandrasekhar 1961; Toomre 1964; Goldreich & Lynden-Bell 1965), a variety of energy sources and sinks (Struck-Marcell & Scalo 1984), and all of the above plus magnetic fields (Elmegreen 1991a, 1994; we will refer to the latter as E94). In particular, the system considered by Elmegreen is nearly identical to the system we consider in our simulations, except that our model is two-dimensional (note also the reversed choice of the x - and y -axes). For the combined instability, E94 gives dispersion relations at $t = 0$ for radial and azimuthal perturbations (see also Elmegreen 1991b). They read, respectively:

$$\omega_R^2 = 2\pi G \sigma k - k^2 \left(\frac{\gamma_{\text{eff}}}{\gamma} c^2 + v_A^2 \right) - \kappa^2, \quad (4a)$$

¹ Note that in Paper I the quantity c_{eff}^2 used in the dispersion relation for gravito-acoustic waves was mistakenly written without the factor $1/\gamma$. This caused an underestimation of the computed periods, which are then in not as good an agreement with the periods observed in the simulations. However, the observed and computed periods still agree within a factor 30%.

$$\omega_A^2 = 2\pi G\sigma k - k^2 \frac{\gamma_{\text{eff}}}{\gamma} c^2 - \frac{\omega_A^2 \kappa^2}{\omega_A^2 + k^2 v_A^2}, \quad (4b)$$

where ω_R (resp. ω_A) is the growth rate in the radial (resp. azimuthal) case, k is the wavenumber, σ is the surface density of the disk, $\kappa = 2\Omega[1 + 1/2 (R/\Omega) d\Omega/dR]^{1/2}$ is the epicyclic frequency, and v_A is the Alfvén velocity. Note that in E94 there is no factor of $1/\gamma$ in the second term of the r.h.s., because in that paper c is defined as the isothermal sound speed, whereas in the present paper it is defined as the adiabatic sound speed. Also, in transcribing eqs. (4) from E94, we have omitted a reduction factor included there to account for the finite thickness of the disk. Note also that in E94 γ_{eff} is *stricto sensu* a different quantity than that used in the present paper. While ours is an *equilibrium* γ_{eff} (denoted $\gamma_{\text{eff}}^{\text{eq}}$), in E94 it is a *perturbation* γ_{eff} (denoted $\gamma_{\text{eff}}^{\text{p}}$), which is obtained from a linear stability analysis of the internal energy equation, treated separately (see also Elmegreen 1991b). However, it can be easily shown that in the limit $\omega \rightarrow 0$, which corresponds to the cooling rate being much faster than the growth rate of the perturbation, $\gamma_{\text{eff}}^{\text{p}} \rightarrow \gamma_{\text{eff}}^{\text{eq}}$. This is precisely the case in our simulations (cf. Paper I). Thus, it is justified to use γ_{eff} as defined in §3.1 in the dispersion relation (4). Incidentally, in the opposite limit, $\omega \rightarrow \infty$, $\gamma_{\text{eff}}^{\text{p}} \rightarrow \gamma$, where γ is the actual heat capacity ratio of the gas. In this case, the growth rate of the perturbation is much faster than the cooling rate, and the flow responds adiabatically.

For our 2D system without shear ($\kappa = 2\Omega$), the dispersion relations for the radial and azimuthal growth rates read, in nondimensionalized form,

$$\lambda_R^2 = J^2 - k^2 \left(\frac{\gamma_{\text{eff}}}{\gamma} T_{\text{eq}} + B_o^2 \right) - \kappa^2, \quad (5a)$$

$$\lambda_A^2 = J^2 - k^2 \frac{\gamma_{\text{eff}}}{\gamma} T_{\text{eq}} - \frac{\kappa^2 \lambda_A^2}{\lambda_A^2 + k^2 B_o^2} \quad (5b)$$

Note that, in contrast to eqs. (4), the gravitational term does not contain the factor k , because of the strict two-dimensionality of our model. When shear is included (κ given by the full expression above), these growth rates cannot be defined any more except for $t \sim 0$.

We now analyze equations (5) in the absence of shear (hence $\kappa \equiv 2\Omega$). In our 2D system, radial perturbations are always stable at all scales when $\tilde{Q}_R = \kappa/J > 1$ (in the nondimensionalized units of the paper, the volume density $\langle \rho \rangle$ is equal to unity), whereas in E94 the criterion reads

$$\tilde{Q}_R = \frac{\kappa (c^2 \frac{\gamma_{\text{eff}}}{\gamma} + v_A^2)^{1/2}}{\pi G \sigma} > 1.$$

On the other hand, in the presence of magnetic fields, azimuthal perturbations are always unstable (albeit slowly if B_o is weak). The unstable wavenumbers, obtained by solving for λ_A^2 in eq. (5b) and requiring $\lambda_A^2 > 0$, are such that $k < J\sqrt{\gamma/(\gamma_{\text{eff}} T_{\text{eq}})}$ (the standard Jeans wavenumber), which clearly can be smaller

than the smallest available wavenumber in the simulation. Thus, for practical purposes, our simulations with magnetic field will be unstable in the azimuthal direction if the unstable scale is smaller than the size of the integration box, i.e., $J^2 > (\gamma_{\text{eff}} T_{\text{eq}}/\gamma)^{1/2}$. A similar condition for radial perturbations leads to $J^2 > (\kappa^2 + B_o^2 + \gamma_{\text{eff}} T_{\text{eq}}/\gamma)^{1/2}$. If this latter condition is satisfied, the medium is then unstable relative to both directions. When there is no magnetic field, the two dispersion relations for the radial and azimuthal directions become identical, and the instability criterion reduces to $J^2 > (\kappa^2 + \gamma_{\text{eff}} T_{\text{eq}}/\gamma)^{1/2}$.

In the presence of shear, the above analysis is valid only for $t \sim 0$. Confirming the results in E94, our numerical simulations indicate that, for low shear rates, the magnetic field opposes radial perturbations which tend to compress the field lines, while it helps azimuthal perturbations since magnetic tension opposes Coriolis spin-up. For high shear, the magnetic field again stabilizes azimuthal perturbations because they are sheared into the radial direction before they have time to collapse (E94). Finally, in the non-magnetic case, there is no distinction between the radial and azimuthal directions, as stated before.

Table 3 shows the results of various simulations aiming at investigating the combined effect of shear, magnetic fields, rotation and γ_{eff} . All runs have $J = .5$. Shown in the Table are the relevant parameters of two sets of runs, without and with magnetic fields. The value of κ indicated in the Table is the minimum over the integration box, as given by the shear profile. The seventh column gives C^2 , an indicator of the linear instability of the runs, which we define as the square of the ratio of the Jeans number to the appropriate rotational or thermal pressure terms as discussed above. The eighth column gives the actual behavior of the simulation, denoting by “S” and “U” stable and unstable runs, respectively. A rough estimate of the actual growth rate of perturbations is given by the time — in units of 10^8 yr — taken by the simulation to reach a peak density of 5 in code units. Note that in Table 3 all runs have $\tilde{Q}_R = \kappa/J > 1$, and are thus stable to radial perturbations. Finally, the column labeled “collapse” gives a description of the type and/or location of the clouds that form.

Note that in the present work, the initial perturbations are Gaussian with random phases, and therefore contain both radial and azimuthal components. A first observation is that many of the runs require a long time before collapsing, consistent with the fact that the growth of the perturbations is not exponential but oscillatory, due to the time-dependent nature of the linearized equations (Elmegreen 1991a). Indeed, a succession of epochs of growth and decay is observed in our simulations before gravitational collapse finally occurs in the unstable runs.

An interesting feature of the functional form we have chosen for the shear (cf. eq. (2)) is that it allows representation of the effects of various amounts of shear in a single simulation. The profile crests have zero shear, while the nodes have maximum shear. Recall that, as mentioned in §2.2, the shear has the same sign as the actual Galactic shear ($d\Omega/dR < 0$) in only the upper and lower quarters of the integration box.

We observe that all non-magnetic runs in Table 3, except run 45, should be

stable according to the linear criterion based on C^2 . However, the linear behavior is seen to be modified by the presence of shear, as exemplified by runs 39 and 105, which differ only by the presence of shear in the former. Although both should be stable, since $C^2 < 1$, run 39 actually forms clouds in a relatively short time. All of these runs exhibit cloud formation in the regions where $d\Omega/dR < 0$ (which minimizes κ), a reflexion of the original linear criterion. Note that the run which exhibits cloud formation on the shortest timescales (run 45) is the only one that is unstable according to the linear criterion. Moreover, this run exhibits clouds formation without preference for specific regions in the integration box because it has $\kappa = 0$.

In the presence of a magnetic field, all runs in Table 3 should be unstable according to the linear criterion, and indeed are. However, the stabilizing effect of the magnetic field in the presence of high shear (E94) can be seen in the fact that the simulations with shear tend to form clouds preferentially in the regions of zero shear. Even though the linear instability criterion does not include the value of the magnetic field, nevertheless it can be observed that runs with larger B_o tend to form clouds more rapidly (compare runs 57 and 47; and runs 56 and 43). This result exemplifies the inhibition of Coriolis spin-up by the magnetic field (magnetic braking). Also, comparing runs 42 and 43, which are identical except for their value of γ_{eff} , we see that the run with zero pressure gradient collapses faster, again a reflection of the original linear criterion. Finally, note that the runs labeled “azimuthal” in the “collapse” column have no shear, leading to a collapse simply along field lines, and forming a radially-oriented cloud.

These results are exemplified in figs. 1a and 1b, in which typical unstable non-magnetic (fig. 1a) and magnetic (fig. 1b) runs are respectively shown at late times in their evolution. In fig. 1a, a large elongated cloud is seen to form in the upper and lower quarters of the domain (recall the boundary conditions are periodic), where $d\Omega/dR < 0$. In fig. 1b, clouds are seen to form at the crests of the shear profile, where $d\Omega/dR = 0$.

Finally, non-magnetic runs with shear (e.g. run 41) initially exhibit an expulsion of material from the central regions of the box, where $d\Omega/dR > 0$, indicating that the classical Rayleigh instability is at work there.

4. MAGNETIC EFFECTS IN THE FULLY TURBULENT REGIME

4.1 *Influence of star formation on magnetic field dynamics*

This section is devoted to a study of the interplay between magnetic field and star formation (SF) in the fully turbulent regime. We will start with the influence that the stellar forcing has on the magnetic field dynamics.

Since our simulations are two-dimensional, we cannot expect any dynamo mechanism. Growth and maintenance of magnetic fluctuations are nevertheless observed in our system. In order to understand the mechanism at play, it is convenient to write the equation for the evolution of the quantity $\mathcal{P} \equiv \int \rho A^2 d^2\mathbf{x}$ where A is the vector potential defined by the relation $\mathbf{B} = \nabla \times (A \mathbf{e}_z)$. After splitting

the magnetic field \mathbf{B} into its constant component $\mathbf{B}_o = B_o \mathbf{e}_x$ and its fluctuating part $\mathbf{b} = \nabla \times (a \mathbf{e}_z)$, we can write an equation for the fluctuating potential

$$\frac{\partial a}{\partial t} = (\mathbf{u} \times \mathbf{B}) \cdot \mathbf{e}_z + \eta(-1)^{n+1} \nabla^{2n} a = -\mathbf{u} \cdot \nabla a + (\mathbf{u} \times \mathbf{B}_o) \cdot \mathbf{e}_z + \eta(-1)^{n+1} \nabla^{2n} a,$$

where $n = 1$ for standard MHD and $n = 4$ in our simulations. Assuming the fields are periodic in the domain or vanish at the boundaries, one obtains for the time evolution of \mathcal{P} :

$$\begin{aligned} \frac{\partial}{\partial t} \int \rho a^2 d^2 \mathbf{x} &= 2 \int \rho a (\mathbf{u} \times \mathbf{B}_o) \cdot \mathbf{e}_z d^2 \mathbf{x} - 2\eta \int \nabla^n (\rho a) \cdot (\nabla^n a) d^2 \mathbf{x} \\ &= -2B_o \int \rho a u_y d^2 \mathbf{x} - 2\eta \int \nabla^n (\rho a) \cdot (\nabla^n a) d^2 \mathbf{x}. \end{aligned} \quad (6)$$

As is well known (Moffatt, 1978) when $B_o = 0$ and ρ is constant, the quantity \mathcal{P} decreases to zero, and thus magnetic fluctuations die away, whatever forcing is applied on the velocity field. When density fluctuations are small enough, the contribution of dissipation to the evolution of \mathcal{P} is again certainly negative. In the general case, one cannot assert if \mathcal{P} will decrease monotonically, although one expects that in general the effect of dissipation will be a cumulative decrease in magnetic energy. Let us now consider the case $B_o \neq 0$. When the system is forced by Alfvén or magnetosonic waves, the first term in the right-hand side of eq. (6) still vanishes and thus magnetic field fluctuations cannot be maintained. This can be seen if one takes for a , u_y and $\delta\rho$ small disturbances of the equilibrium state $a = 0$, $u = 0$, $\rho = 1$, proportional to the eigenvectors of the linearized equations. When these disturbances have an oscillatory behavior (stable case), $\delta\rho$ and u have the same parity, opposite to that of a , so that $\int (1 + \delta\rho) a u d^2 \mathbf{x} = 0$. When these disturbances grow or decay on the other hand (unstable case), a and u have the same parity, opposite to that of $\delta\rho$, so that the first part of the integral, $\int a u d^2 \mathbf{x}$, is nonzero. The first term on the r.h.s. of eq. (6) is positive and can thus balance or even overcome dissipation. It is precisely the situation when a single cloud contracts or relaxes. The fields ρ , a , u_y are schematically depicted in fig. 2a for a magnetosonic wave and in fig. 2b for a cloud contraction. This mechanism is probably as efficient as a dynamo to sustain magnetic fluctuations, but a dynamo is still needed to generate the locally constant field B_o (the constant mode B_o is dynamically disconnected from the other modes).

In order to test numerically the mechanism for the generation of magnetic fluctuations, we have performed the series of runs denoted 12, 13, 14 in Table 2 for which the initial fluctuations of the magnetic field are set to $\delta B = 0.01$ and the constant field B_o takes respectively the values 0, 0.03 and 0.3. Figure 3 displays the fluctuating magnetic energy as a function of time for the three runs. It is very clear that we indeed have a growth of $\int (\delta B)^2 d^2 \mathbf{x}$ when $B_o \neq 0$. The values of $\int (\delta B)^2 d^2 \mathbf{x}$ reached during the run are also increasing with B_o . The fluctuating magnetic energy

and the density fluctuations are correlated when B_o is nonzero, whereas they are not when $B_o = 0$ (not shown).

A few additional points are worth noting concerning the constant field B_o . First, its presence is a regularizing factor in inviscid incompressible MHD (Bardos, Sulem & Sulem, 1988). Second, it also removes neutral points (probably not all of them if it is too weak) and thus leads to a decrease of magnetic dissipation. Finally, it has also been shown that the presence of a constant field B_o hinders small scale turbulence (Shebalin, Matthaeus & Montgomery, 1981). In compressible MHD turbulence permeated by a uniform magnetic field, the solenoidal enstrophy is much smaller than when $B_o = 0$, whereas its compressible counterpart or density fluctuations remain unaffected (A. Broc, 1993).

In every MHD simulation, we observe a tendency for the typical scale of the magnetic fluctuations to grow in time. This effect can be observed in figs. 4a and 4b, which respectively display the magnetic spectra at times $t = .65 \times 10^8$ and $t = 2.6 \times 10^8$ yr for run 15. Note the growth of the $k = 1$ mode for the b_y component. A study in three dimensions would be required to test whether this mechanism persists or is a consequence of the two-dimensional assumption. But let us mention that in the three-dimensional incompressible case, there is an inverse cascade of magnetic helicity, with the magnetic energy following to a lesser extent (Pouquet, Frisch & L  orat, 1976; Meneguzzi, Frisch & Pouquet, 1981; Horiuchi & Sato, 1986, 1988).

Figure 5 also displays spectra of the total magnetic energy together with those of the solenoidal and compressive components of the velocity field. Note that the magnetic spectrum is almost in equipartition with the solenoidal spectrum, as already observed in two-dimensional compressible MHD simulations (Pouquet, Passot & L  orat, 1991; Dahlburg & Picone, 1989; Picone and Dahlburg, 1991). Contrary to the case of a decaying MHD compressible turbulence, we do not observe a systematic growth of $\mathbf{u} \cdot \mathbf{B}$ correlations. On the other hand it is quite clear that for finite but moderate values of B_o , this correlation oscillates noticeably, the extrema increasing with B_o . When B_o is zero and the wavenumber of magnetic fluctuations k_B is initially equal to 4, the correlation actually goes steadily to zero. The temporary growth of $\mathbf{u} \cdot \mathbf{B}$ correlations is probably due to a transfer of magnetic energy created by clump contraction, to large scale Alfv  n waves along B_o . These results are markedly different when B_o is very large because of a strong confinement of the clouds (see below). A phenomenon which is also common to our present and former MHD simulations (Pouquet et al. 1991), is that *the global compressibility of the medium (as measured by the ratio of compressible to total kinetic energy) increases with the intensity of the fluctuating magnetic energy.*

It is interesting to note that the topology of the magnetic field, mainly parallel to the local shear, is actually enforced by the shear itself. A run with the uniform component of the magnetic field initially perpendicular to the direction of the shear, evolves so that the magnetic field becomes aligned with the shear profile.

The field topology is almost uniform in intercloud regions and very turbulent in cloud complexes undergoing star formation (fig. 6). The magnetic field has

actually an important dynamical effect in these regions, favoring clumpiness and fragmentation. On the other hand, at its fiducial value, it seems rather inefficient in intercloud regions to influence cloud motions. We indeed observe clouds propagating almost in any direction with respect to the magnetic field. The turbulent character of the magnetic field inside clouds is also possibly at the origin of the lack of correlation between B^2 and ρ in these regions, as indicated by fig. 7b which shows a scatter plot of the square of the magnetic field strength as a function of density for the complex encircled in fig. 7a.

Finally, the typical values of the field strength within clouds is $\sim 12\mu\text{G}$, but can possibly reach $30\mu\text{G}$. This can be compared to the value in the ICM of $\sim 3\mu\text{G}$. As opposed to quasi-stationary models, which cannot reach such high contrasts between cloud and intercloud field strengths (Mc Kee et al. 1993), the present turbulent model is capable of producing realistic values of the magnetic field strength within clouds.

4.2 Influence of the magnetic field on cloud and star formation

The second part of this section is devoted to the influence of magnetic fields on the formation of density clumps (eventually leading to star formation). As in the linear regime, a constant field B_o hinders cloud condensations and thus star formation when it is small (super-Alfvénic motions) but favors it when it is large (sub-Alfvénic motions). In the case of even much larger values of B_o , star formation gets inhibited again. These results are summarized in fig. 8 which displays the total star formation (SFR integrated over the total lapse time of the simulations) for runs with different values of the constant magnetic field B_o but otherwise identical (runs 53, 6, 19, 54, and 61 of Table 2). If the uniform field B_o is relatively weak, clumps get significantly disrupted by the shear – originating either from differential rotation or from turbulence – before they can contract along the magnetic field; further contraction, that has to occur across the magnetic field, is then hindered by magnetic pressure. On the other hand if the uniform field is relatively strong, it can act against the stabilizing action of the Coriolis force through the magnetic braking mechanism, and clump contraction can then occur along the field before shear can disrupt the condensation. This trend is valid even when the magnetic field fluctuations are at smaller scale (see runs 11, 15 and 5 in Table 2).

For even larger values of B_o , another effect comes into play that we call the “pressure cooker” effect. Shells formed by stellar heating cannot expand as much when the magnetic field is strong, resulting in confining clouds in complexes or very thick filaments. Inside these complexes, star formation induces a strongly turbulent state where smaller roundish clouds get formed (see fig. 9 for run 61). Figure 10 also shows this effect by comparing three runs with different values of the initial uniform magnetic field component (runs 53, 19 and 61 in Table 2). However, with such an inhibition of shell expansion, stellar activity is *globally* reduced (self-propagating SF is not as effective); also reduced are both the compressibility and the amount of $\mathbf{u} \cdot \mathbf{B}$ correlations since Alfvén waves cannot get efficiently excited. Now, the cloud complexes have a tendency to follow the shear. This “pressure cooker” effect is

present even at smaller values of B_0 although it is less obvious. Expansion motions tend to amplify magnetic fluctuations perpendicular to the velocity, and as a result these motions get decelerated due to the influence of the tension of the amplified field lines. In these regions, the field lines are perpendicular to the density gradients, i.e. parallel to the density features (fig. 6).

In general, one can say that the effects of a large uniform field are somewhat opposite to those of a large fluctuating field.

Planar shocks propagating perpendicular to a constant magnetic field have a reduced compressibility, compared to the non-magnetic case, and if the pre-shock state is at rest, the shock must propagate at $U_s > (c^2 + v_A^2)^{1/2}$. If the field gets amplified beyond a certain value due to nonstationary expanding motions, the shock will not be able to propagate and will give rise to a magnetosonic wave. The shell will then stop to propagate as an “entity”, and to collect matter on its path, and the motion of the resulting cloud will become a wave propagation. In the simulations we observe that shells propagate at speeds of the order of 8 km s^{-1} while the typical sound speed in the ICM is $\sim 12 \text{ km s}^{-1}$ and the Alfvén speed $\sim 30 \text{ km s}^{-1}$. Other effects take place when the field is curved. As mentioned above, the field line tension tends to oppose expanding motions, while the Lorentz force, which is radial and oriented inward, promotes converging motions and generates blobs. As a result, planar structures are less likely to subsist in the magnetic case and we actually observe more roundish structures when the magnetic field is stronger and/or at smaller scales. Also, an expanding shell will break up more easily in presence of a magnetic field due to the Rayleigh-Taylor (and possibly the Parker) instability.

Another effect of the magnetic field is linked to the scale of its fluctuations. Star formation is reduced when the field fluctuations are on smaller scale (compare runs 53, 19 and 54 with runs 11, 15 and 5 in Table 2). This effect is the result of the magnetic pressure acting at a comparable scale as the one of the fluid motions leading to clump contraction. In a run initiated with $\mathbf{k}_B = 4$ (as opposed to the standard case with $\mathbf{k}_B = 1$), the inverse cascade mechanism gradually enhances the large-scale magnetic field, and consequently star formation increases with time in this run.

Star formation is also proportional to the intensity of the magnetic fluctuations δB , the more so when these are on large scale, because in that case, they locally act as a constant magnetic field (compare runs 8 and 7 with runs 19 and 5 in Table 2).

Before closing this section, in summary we emphasize that in the turbulent runs, even though we partly recover the general trends of the linear stability analysis, it is however clear that the formation, dynamics and morphology of clouds are dominated by turbulent energy injected from stars, rather than by the combined linear instability.

5. GENERAL BEHAVIOR OF A FULLY TURBULENT SIMULATION

In this section we briefly describe some noteworthy features of run 28, a

simulation with fiducial values of the parameters, using 512×512 grid points (see Table 2). According to our choice of parameters, one pixel corresponds to a size of roughly 2 pc. Although our viscosity is kept at the minimum compatible with the stability of our numerical scheme, it damps velocity features at scales less than about 10 pc.

Figures 11a, b and c respectively show the density, pressure and temperature for this run at time $.72 \times 10^8$ yr. Other views of this run at different times are given in figs. 6 and 7a. In fig. 11a, we identify structures that can be called “giant complexes”, with sizes of several hundreds of pc, “complexes”, with sizes ~ 100 pc, and “clouds”, with sizes of a few tens of pc. However, we stress that this classification is rather arbitrary, since in reality there is a continuum of density structures that can be identified by thresholding the density at continuously varying levels. This fact led us in Paper I to use a continuous filling factor function. For the MHD simulations, this point and statistical properties of the clouds will be discussed in a future paper.

One of the most striking features of the density structures in the simulation is their amorphous character, none of the clouds being nearly circular, nor even elliptical. Moreover, density structures are hierarchically nested, as observed in real interstellar clouds (see, e.g., Scalo 1985). This phenomenon was also observed in non-magnetic simulations (Vázquez-Semadeni, 1994, Paper I).

Extremely long and thin filaments of density and pressure (with low contrasts) are observed. The filaments are correlated with jumps of the velocity and magnetic field (compare figs. 11a and b with the magnetic field components shown in figs. 12a and 12b). This is also apparent in both the vorticity and the current (not shown). Similar filamentary structures obtain in the non-MHD runs of Paper I, as well as in the simpler non-rotating decay runs of compressible turbulence (Passot et al., 1988).

Star-forming (“HII”) regions have sizes of a few tens of pc, thus corresponding to the largest observed sizes for HII regions in real galaxies. In our simulations, these regions cannot be much smaller than this because of the Gaussian smoothing used for numerical reasons (Paper I, §2.2). “HII” regions in the simulations are most easily observed in the pressure and temperature images, as seen in figs. 11b and 11c. A particularly active star-forming region is seen in fig. 11b near the upper-right corner, and in fact this region is somewhat evocative of images of the neighborhood of 30 Dor in the Large Magellanic Cloud.

Global gravitational contraction of a giant complex appears to occur in the upper right quadrant, as can be seen in the density field for run 28 at times $t = .42 \times 10^8$ yr (fig. 6), $t = .72 \times 10^8$ yr (fig. 11a), and $t = .91 \times 10^8$ yr (fig. 7a). However, generalized collapse is halted by SF activity, which increases the turbulence in the highest-density regions of the complex. Also, the kinetic and magnetic energies in various clouds appear to be within a factor of 5 from each other, suggesting rough equipartition. A detailed study of the energetics of the clouds and complexes in run 28 will be presented in a future paper.

6. CONCLUSIONS

6.1. *Summary*

In this paper we have presented an extension of the model introduced in Paper I, now incorporating magnetic fields, Galactic disk rotation and a density-dependent diffuse heating which results in a softer equation of state. The behavior of the model was studied both in the linear and nonlinear regimes. In the linear regime, the model is well described by the linear theory developed by Elmegreen (1991a, E94; see also Elmegreen 1991b). At low shear, a weak field stabilizes the medium by opposing collapse of radial perturbations, while a strong field is destabilizing by preventing Coriolis spin-up of azimuthal perturbations (magnetic braking). At high shear, azimuthal perturbations are sheared into the radial direction before they have time to collapse, and the magnetic field becomes stabilizing again. In the absence of magnetic field, the problem reduces to the classical Toomre (1964) criterion, slightly modified for our 2D model.

In the nonlinear regime, a variety of interesting effects are present, many of them unforeseeable through linear analyses:

An amplification mechanism for the fluctuating magnetic field has been identified, that allows the maintenance of magnetic energy over the long time evolution of the interstellar medium. It requires the presence of a constant component B_0 and is effective for contracting or expanding motions, such as the expanding shells generated by stellar heating.

The fluctuating magnetic field is generated at the scale of a cloud and is then transferred to larger scales.

The magnetic field dynamics clearly does not reduce to the propagation of waves. The field participates to the global magneto-hydrodynamic turbulence, as demonstrated by the magnetic spectra, which exhibit a developed inertial range.

The importance of the magnetic field on the global dynamics is complex. It depends both on its topology and its intensity compared to the shearing motions. A strong magnetic field confines matter in big complexes where smaller roundish clumps are formed, globally inhibiting star formation. The field is turbulent inside the clouds and straight in intercloud regions.

No unique correlation is found between the density and the magnetic field intensity probably due to its turbulent character. However, its orientation at the edge of the clouds tends to be perpendicular to the density gradient. Also, the magnetic field strength within clouds is roughly a factor of 4 larger than in the ICM with excursions of up to factors of 10. This suggests that the field in clouds is amplified through flux freezing by the same collisions of gas streams that form the clouds (Hunter et al. 1986; Elmegreen 1993; Paper I).

The velocity of the expanding shells rapidly becomes sub-Alfvénic.

6.2. *Discussion of limitations*

Some of the most important limitations of this work are the relatively low effective resolution, the two-dimensionality of the simulations, and the omission of

supernovae. In this section we briefly discuss their possible consequences.

As mentioned in §5, viscosity damps velocity features at scales smaller than 3–5 pixels (~ 6 – 10 pc at a resolution of 512 grid points per dimension). Additionally, the mass diffusion smooths out density fluctuations at comparable scales. These scales correspond to those of sizable molecular clouds. Thus, the simulations are incapable of resolving the structure within the clouds of those sizes, the applicability of the model being restricted to structures ranging from superclouds to large individual molecular clouds. However, the usage of a hyperviscosity scheme with a high power of the Laplacian guarantees that features in this range of scales are virtually unaffected by viscosity, and are thus fully turbulent. Moreover, note that in this paper we have focused on problems that do not require a very high resolution to deal with, as evidenced by the fact that only one run of those reported in Table 2 has a resolution of 512 grid points per dimension, while all others have 128.

The two-dimensionality of the simulations has several effects. Among them are the absence of vortex stretching, and the modification of the gravitational potential leading to a force proportional to the inverse of the distance between clouds. Other effects are also mentioned in Vázquez-Semadeni (1994) and Paper I. Moreover, a dynamo effect is obviously absent from our simulations. However, we have identified an alternative mechanism by which expanding bubbles can amplify the magnetic energy through amplification of the magnetic potential, provided a uniform component of the field is present.

Another important consideration is that the two-dimensionality of the simulations implies the existence of an inverse (from small to large scales) energy cascade which is not present in three dimensions (Kraichnan 1967), and therefore a reduced rate of energy dissipation at small scales compared to the 3D case (see, e.g., Lesieur 1990). Note, however, that this does not lead, for example, to excessively turbulent regimes. In purely-hydrodynamic incompressible turbulence with random extended forcing, the inversely-cascading energy organizes itself into large-scale vortex pairs which exist within a normally turbulent background (McWilliams 1984). Furthermore, in the case of our compressible simulations with a pointwise compressible forcing, the latter (stellar heating) produces expanding shells that create turbulence at large scales, although without the formation of large-scale vortices. This process is different from an inverse cascade and must be present in 3D as well as in 2D, decreasing the difference between the two cases.

Supernovae (SNe) are omitted from the present work in part because new code development would be required. It is not clear whether their presence will have (or not) a sizable effect in particular on the density and temperature of the ICM. Such an omission results in the absence in our models of a hot ($T \sim 3 \times 10^5$ – 10^6 K, $\rho \sim 10^{-2}$ cm $^{-3}$) gas phase. Under the classical picture of McKee & Ostriker (1977), this hot gas occupies the vast majority of the volume (filling factor ~ 1), and in this framework our warm ICM ($T \sim 10^4$ K, $\rho \sim 10^{-1}$ cm $^{-3}$) ought to be essentially replaced by the hot gas. On the other hand, recent numerical results of Slavin & Cox (1992, 1993) suggest that the filling factor of the hot gas may be as low as 20%. In this case, our results would only be modified by the inclusion of mostly

isolated bubbles of hot gas with a relatively small filling factor. Which view is more realistic is currently a matter of debate, and depends on a variety of issues, such as whether a supernova remnant breaks out of its high-density surroundings before it has had time to cool off (e.g., Mac Low & McCray 1988; also G. García-Segura, private communication), and whether the magnetic field forces the surrounding shell to rebound (Slavin & Cox 1992). Our finding that a “pressure cooker” effect reduces shell expansion for our model HII regions seems to support the Slavin & Cox picture, but the degree to which the low-density gas in our simulations will change upon introduction of SNe cannot be assessed precisely. The simulations of Rosen & Bregman (1994) do include SNe and contain a sizable fraction of hot gas, but since their SF scheme is smooth in space and time, significant differences may arise upon consideration of a more realistic, spatially-discrete, thresholded SF scheme like the one used in the present paper. Moreover, their calculations do not include the magnetic field.

In any case, in spite of our neglect of SNe, it is likely that the morphology and energetics *of the clouds* will not be dramatically affected even if the hot phase is pervasive, as its pressure should be comparable to that of our warm ICM and the dynamics induced by expanding shells is already present in our simulations and with comparable total kinetic energies (note that the total kinetic energy inputs to the ISM from OB ionizing radiation, winds, and from SNe are of comparable magnitude (Abbott 1982; García-Segura, Mac Low and Langer 1995)). On the other hand, the inclusion of forcing from SNe and OB winds may result in the medium being even more turbulent, and the mechanism of cloud formation by turbulent density fluctuations exemplified in Paper I might become even more important. Work considering these effects is in progress (Gazol et al. 1995), in which we will address questions such as the filling factors of the various gas phases in the midplane of the Galactic disk and the longevity of the 10^6 K gas.

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REFERENCES

- Abbott, D. C. 1982, ApJ, 263, 723
 Babiano, A., Basdevant, C., Legras, B., & Sadourny, R. 1987, J. Fluid Mech., 183, 379
 Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton: Univ. Press)

- Canuto, C., Hussaini, M. Y., Quarteroni, A., & Zang, T. A. 1988, *Spectral Methods in Fluid Dynamics* (Berlin: Springer-Verlag)
- Chandrasekhar, S. 1961, *Hydrodynamic and Hydromagnetic Stability* (London: Oxford University Press), p. 589
- Crutcher, R. M., Kazes, I., & Troland, T. H. 1987, 1987, *A&A*, 181, 119
- Dalgarno, A., & McCray, R. A. 1972, *ARAA*, 10, 375
- Elmegreen, B. G. 1991a, *ApJ*, 378, 139
- Elmegreen, B. G. 1991b, in *The Physics of Star Formation and Early Stellar Evolution*, ed. C. J. Lada & N. D. Kylafis (Dordrecht: Kluwer), p. 35
- Elmegreen, B. G. 1993, *ApJ*, 419, L29
- Elmegreen, B. G. 1994, *ApJ*, 433, 39
- Falgarone, E., & Puget, J. L. 1986, *A&A*, 162, 235
- Falgarone, E., Puget, J.-L. & Péroult, M. 1992, *A&A*, 257, 715
- Franco, J., & Cox, D. 1986, *PASP*, 98, 1076
- Gammie, C. 1994, *ApJ* (submitted)
- García-Barreto, J. A., Burke, B. F., Reid, M. J., Moran, J. M., Haschick, A. D., & Schilizzi, R. T. 1988, *ApJ*, 326, 954
- Goldreich, P. & Lynden-Bell, D. 1965, *MNRAS*, 130, 7
- Goodman, A. A. 1991, in *Atoms, Ions and Molecules: New Results in Spectral Line Astrophysics*, ed. A. D. Haschick & P. T. P. Ho (San Francisco: ASP), 333
- Heiles, C. 1990, *ApJ*, 354, 483
- Heiles, C., Goodman, A. A., McKee, C. F., & Zweibel, E. G. 1993, in *Protostars and Planets III*, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. of Arizona Press), 279
- Kutner, M. L. 1994, *BAAS*, 26, 1397
- Mac Low, M.-M. & McCray, R. 1988, *ApJ*, 324, 776
- McKee, C. F., & Ostriker, J. P. 1977, *ApJ*, 218, 148
- McKee, C. F., Zweibel, E. G., Goodman, A. A., & Heiles, C., in *Protostars and Planets III*, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. of Arizona Press), 327
- McWilliams, J. 1984, *J. Fluid Mech.*, 146, 21
- Meneguzzi, M., Frish, U., & Pouquet, A. 1981, *Phys. Rev. Lett.* 47, 1060
- Mouschovias, T. 1976a, *ApJ*, 206, 753
- Mouschovias, T. 1976b, *ApJ*, 207, 141
- Myers, P. C., & Goodman, A. A. 1988a, *ApJ*, 326, L27
- Myers, P. C., & Goodman, A. A. 1988b, *ApJ*, 329, 392
- Pouquet, A., Passot, T., & Léorat, J. 1991, in *Fragmentation of Molecular Clouds and Star Formation*, IAU Symp. 147, ed. E. Falgarone, F. Boulanger & G. Duvert (Dordrecht: Kluwer), 101
- Pudritz, R. E., & Gómez de Castro, A. I. 1991, in *Fragmentation of Molecular Clouds and Star Formation*, ed. E. Falgarone, F. Boulanger, & G. Duvert (Dordrecht:Kluwer), 317
- Rand, R. J., & Kulkarni, S. R. 1989, *ApJ* 343, 760
- Raymond, J. C., Cox, D. P., & Smith, B. W. 1976, *ApJ* 204, 290
- Rosen, A., Bregman, J. N., & Norman, M. L. 1993, *ApJ*, 413, 137

- Rosen, A., & Bregman, J. N. 1994, BAAS, 25, 1394
 Shore, S. N. 1989, in Encyclopedia of Astronomy and Astrophysics, ed. R. A. Meyers (San Diego: Academic Press), 183
 Shu, F. 1992, Gas Dynamics (Mill Valley: University Science Books)
 Slavin, J. & Cox, D. 1992, ApJ, 417, 187
 Slavin, J. & Cox, D. 1993, ApJ, 417, 187
 Struck-Marcell, C. & Scalo, J. 1984, ApJ, 277, 132
 Toh, S., Ohkitani, K., & Yamada, M. 1991, Physica D, 51, 569
 Toomre, A. 1964, ApJ, 139, 1217
 Trimble, V. 1990, in Galactic and Intergalactic Magnetic Fields, ed. R. Beck, P. P. Kronberg, & R. Wielebinski (Dordrecht: Kluwer), 29
 Troland, T. H. 1990, in Galactic and Intergalactic Magnetic Fields, ed. R. Beck, P. P. Kronberg, & R. Wielebinski (Dordrecht: Kluwer), 293
 Vázquez-Semadeni, E., Passot, T. & Pouquet, A. 1995, ApJ
 Wielebinski, R., & Krause, F. 1993, A&A Rev., 4, 449
 Zweibel, E. 1987, in Interstellar Processes, ed. D. J. Hollenbach & H. A. Thronson (Dordrecht: Reidel), 195

FIGURE CAPTIONS

Fig. 1. a) Contour plot of the density field of run 39, a non-magnetic run in the linear regime, at $t = 2.47 \times 10^8$ yr. Ω points upwards perpendicular to the plane of the page. A large cloud has formed in the region where $d\Omega/dy < 0$ (the upper and lower quarters of the domain; recall the boundary conditions are periodic). Maximum density for this plot is $\rho_{\max} \sim 5$. After this time, this cloud still goes through several oscillations before collapsing. b) Contour plot of the density field of run 43 at $t = 2.61 \times 10^8$ yr. This is a magnetic run, also in the linear regime. In this case, clouds form at the crests of the shear profile given by eq. (2), where $d\Omega/dy = 0$. Maximum density for this plot is $\rho_{\max} \sim 3$.

Fig. 2. Sketches of the density ρ (*dash-dotted line*), velocity u (*long-dashed line*) and magnetic potential a (*solid line*) for a magneto-sonic wave (*top*) and for a cloud compression (*bottom*) along the y -axis, perpendicular to B_o . For waves, ρ and u have the same parity, opposite to that of a , whereas for the compression a and u have the same parity, opposite to that of ρ .

Fig. 3. Evolution of the fluctuating component $(\mathbf{B} - \mathbf{B}_o)^2$ of the magnetic field for three runs with different initial values of the uniform component of the magnetic field B_o . *Solid line*: run 12, $B_o = 0$. *Dotted line*: run 13, $B_o = .15 \mu\text{G}$. *Dashed line*: run 14, $B_o = 1.5 \mu\text{G}$. Both the amplitude of the fluctuations and their time derivative are seen to increase with B_o .

Fig. 4. Magnetic spectra of run 15 (initial magnetic centroid $k_B = 4$) at two different times: $t_8 = 0.65 \times 10^8$ yr (*top*) and $t_8 = 2.6 \times 10^8$ yr (*bottom*). Solid lines correspond to B_x and dotted lines to B_y . Note that the energy in both components

of the magnetic field is transferred to lower modes as time increases (particularly so for the B_y component).

Fig. 5. Velocity and magnetic spectra for run 28 at $t = 2.6 \times 10^8$ yr. *Solid line*: total magnetic spectrum. *Dotted line*: solenoidal velocity spectrum. *Dashed line*: compressible velocity spectrum. Note that the magnetic and the solenoidal spectra are within factors of a few from each other at all scales.

Fig. 6. Density and magnetic fields for run 28 at $t = 0.42 \times 10^8$. The density grey scale is logarithmic, and saturates at $\rho = 40 \text{ cm}^{-3}$. The arrow at the bottom right indicates a field strength of $30 \mu\text{G}$. Note the strong magnetic turbulence inside clouds and the rather smooth character of the magnetic field in the ICM. Regions of alignment of the magnetic field and density features can be seen in the filament in the upper right corner. However there are also regions where the magnetic field is perpendicular to the density features such as the lower portion of the same cloud and also the cloud near the center of the lower left quadrant.

Fig. 7. (a) Density field for run 28 at $t = 9.1 \times 10^7$ yr showing a circular complex of radius 30 pixels ($\sim 60\text{pc}$). The density grey scale is as in fig. 6. (b) B^2 vs. ρ for the circular region of fig. 7a. Spherically symmetric compressions should give a linear relationship. The scatter diagram indicates that there is no preferred compression geometry.

Fig. 8. Time integral of the star formation (SF) rate for various runs as a function of the initial value of the uniform component of the magnetic field B_o . From smaller to larger values of B_o , the points represent runs 53, 6, 19, 54 and 61. At small B_o , SF is inhibited because B_o is small enough not to counteract magnetic braking, but is able to prevent radial collapse of sheared condensations. Intermediate values of B_o counteract magnetic braking and thus promote SF. Very large values of B_o inhibit SF again because the magnetic field rigidifies the medium.

Fig. 9. Contour plot of the density field for run 61 with magnetic field vectors superimposed. The uniform magnetic field strength B_o for this run is $10\mu\text{G}$. At this value of B_o (about six times the fiducial value), the magnetic field is almost unperturbed by the fluid motions but the clouds exhibit more roundish shapes. The magnetic field fluctuations, even though they are small compared to B_o , occur mainly inside the clouds.

Fig. 10. Contour plots of the density field at $t = 5.2 \times 10^6$ yr for three runs with progressively larger values of B_o , but otherwise identical. From left to right, run 53 ($B_o = 0$), run19 ($B_o = 1.5\mu\text{G}$), and run 61 ($B_o = 10\mu\text{G}$). Note the tendency towards more roundish structures as B_o increases.

Fig. 11. Grey scale images of (a) density, (b) pressure and (c) temperature fields of run 28 at $t = .72 \times 10^8$ yr. The bright spots within the large cloud complex in the upper right are star-forming regions, or “HII regions”. The grey scale in a) and b) is logarithmic. The density grey scale saturates at $\rho = 40 \text{ cm}^{-3}$ and the pressure scale saturates at $P = 5$ in code units ($8.3 \times 10^4 \text{ cm}^{-3} \text{ K}$). Note the globally low pressure

contrast, with typical intercloud values ranging between 3,300 and 6,700 cm^{-3} K, and typical cloud values around 10,000 cm^{-3} K. Exceptions are the “HII” regions, where the pressure reaches $1.8 \times 10^5 \text{ cm}^{-3}$ K, and which are strongly saturated in this image. In c), the grey scale is linear and spans a temperature range of 0 to 20,000 K.

Fig. 12. Grey scale images of the x - (a) and y - (b) components of the magnetic field for run 28 at $t = .72 \times 10^8$ yr. The grey scale ranges from -7.5 to $7.5 \mu\text{G}$. The field is most turbulent in complexes and clouds. Note the discontinuities in both components of the field, which are coincident with the long, thin filaments observed in the density and pressure fields (figs. 11a and b).